Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. In a right $\triangle$ABC, we have: $\sin^2 A + \sin^2 B + \sin^2 C$ = ? **[1]**

a) 2 b) 3

c) 0 d) 1

2. Which one of the following is not a function? **[1]**

a) $\{(x, y) : x, y \in R, y = x^3\}$ b) $\{(x, y): x, y \in R, x^2 = y\}$

c) $\{(x, y) : x, y \in R, y^2 = x\}$ d) $\{(x, y) : x, y \in R, x = y^3\}$

3. If P(E) denotes the probability of an event E, then E is called certain event, if **[1]**

a) P(E) is either 0 or 1 b) P(E) = 0

c) P(E) = 1 d) P(E) = $\frac{1}{2}$

4. $\lim_{x \to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} =$ **[1]**

a) does not exist b) 0

c) $-\frac{1}{3}$ d) $\frac{1}{3}$

5. Equation of the line passing through (1, 2) and parallel to the line y = 3x - 1 is **[1]**

a) y - 2 = 3 (x – 1) b) y + 2 = 3 (x + 1)

c) y + 2 = x + 1 d) y – 2 = x – 1

6. For two sets $A \cup B = A$ if **[1]**

a) A = B b) $A \neq B$

c) B ⊆ A

d) A ⊆ B

7. If $z = \frac{1}{(2+3i)^2}$, then |z| = **[1]**

a) $\frac{1}{13}$

b) $\frac{1}{5}$

c) None of these

d) $\frac{1}{12}$

8. The domain of the function f defined by f(x) = $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$ is equal to **[1]**

a) $(-\infty, -1) \cup [1, 4)$

b) $(-\infty, -1] \cup (1, 4)$

c) $(-\infty, -1) \cup (1, 4]$

d) $(-\infty, -1) \cup [1, 4]$

9. Solve the system of inequalities (x + 5) - 7(x - 2) $\geq$ 4x + 9, 2(x - 3) - 7(x + 5) $\leq$ 3x - 9 **[1]**

a) $\frac{-9}{4} \leq x \leq 1$

b) $-4 \leq x \leq 1$

c) $-1 \leq x \leq 1$

d) $-4 \leq x \leq 4$

10. $\frac{\sin 3x}{1+2\cos 2x}$ is equal to **[1]**

a) $-\frac{1}{2}\cos 2x$

b) sin x

c) 0

d) $\frac{1}{2}$

11. Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then A ∪(B ∩ C) is **[1]**

a) {1, 2, 3}

b) {3}

c) {1, 2, 3, 4, 5, 6}

d) {1, 2, 3, 4}

12. Sum of an infinitely many terms of a G.P. is 3 times the sum of even terms. The common ratio of the G.P. is **[1]**

a) 2

b) $\frac{3}{2}$

c) none of these

d) $\frac{1}{2}$

13. $(\sqrt{3}+1)^{2n} + (\sqrt{3}-1)^{2n}$ is **[1]**

a) negative real number

b) an even positive integer

c) an odd positive integer

d) irrational number

14. If x < 7 , then **[1]**

a) -x > -7

b) -x $\geq$ -7

c) -x < -7

d) -x $\leq$ -7

15. If A and B are two sets then $A \cap (A \cap B') = \ldots.$ **[1]**

a) ∈

b) A

c) ϕ

d) B

16. A railway train is moving on a circular curve of radius 1500 m at a speed of 90 km/hr. Through what angle has it turned in 11 seconds? **[1]**

a) 10° 30'

b) 12°

c) 16°30'

d) 11°40'

17. If $\frac{(a^2+1)^2}{2a-i}$ = x + iy, then $x^2 + y^2$ is equal to **[1]**

a) $\frac{(a^2+1)^4}{4a^2+1}$

b) $\frac{(a^2-1)^2}{(4a^2-1)^2}$

c) None of these

d) $\frac{(a+1)^2}{4a^2+1}$

18. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is **[1]**

a) $^{20}C_9$

b) $^{16}C_{11}$

c) $^{16}C_5$

d) $^{16}C_9$

19. **Assertion (A):** The expansion of (1 + x)^n = $n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If each of the observations x_1, x_2, ..., x_n is increased by a, where a is a negative or positive number, then the variance remains unchanged. **[1]**

Reason (R): Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the domain and the range of the real function, $f(x) = \frac{1}{\sqrt{x+|x|}}$. **[2]**

OR

If A = {1, 3, 5}, B = {x, y} represent the product by arrow diagram: B $\times$ B.

22. Differentiate: $\frac{(x^2-1)}{(x^2+7x+1)}$. **[2]**

23. Find the locus of the centre of the circle touching the line x + 2 = 0 and x - 2y = 0. **[2]**

OR

If the parabola y^2 = 4ax passes through the point (3, 2), then find the length of its latusrectum.

24. Let A = {a, b, c, d}, B = {a, b, c} and C = {b, d}. Find all sets X such that: X $\subset$ A and X $\not\subset$ B. **[2]**

25. Put the equation $\frac{x}{a} + \frac{y}{b}$ = 1 to the slope intercept form and find its slope and y-intercept. **[2]**

Section C

26. Let X = {1, 2, 3, 4, 5, 6, 7, 8, 9}. Let R_1 be a relation on X given by R_1 = {(x, y): x - y is divisible by 3} and R_2 be another relation on X given by R_2 = {(x, y) : {x, y) $\subset$ {1, 4, 7} or {x, y} $\subset$ {2, 5, 8} or {x, y} $\subset$ {3, 6, 9}}. Show that R_1 = R_2. **[3]**

27. Solve $\frac{1}{2-|x|} \geq 1, x \in R - [-2, 2]$. **[3]**

28. Show that the points A(1, -1, -5), B(3, 1, 3) and C (9, 1, -3) are the vertices of an equilateral triangle. **[3]**

OR

Three vertices of a parallelogram ABCD are A(3, -1, 2), B(1, 2, -4) and C(-1, 1, 2). Find the coordinates of the fourth vertex.

29. Expand the given expression $\left(x + \frac{1}{x}\right)^6$ **[3]**

OR

Expand the given expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

30. Find the square root of 1 - i. **[3]**

OR

Evaluate $\left[\frac{1}{1-4i} - \frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]$ to the standard form.

31. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to (i) chemical C_1 but not chemical C_2 (ii) Chemical C_2 but not chemical C_1 (iii) Chemical C_2 or chemical C_1. **[3]**

Section D

32. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that all the three balls are red. **[5]**

33. Differentiate the function with respect to x using first principle: cos $(x^2 + 1)$ **[5]**

OR

Evaluate the following limits: $\lim\limits_{x \to \sqrt{6}} \frac{\sqrt{5+2x} - (\sqrt{3}+\sqrt{2})}{x^2-6}$.

34. The ratio of A M and G. M of two positive no. a and b is m : n show that **[5]**
$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

35. Prove that: $\cos 36^o \cos 42^o \cos 60^o \cos 78^o = \frac{1}{16}$. **[5]**

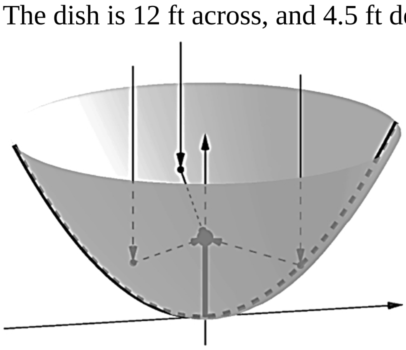
OR

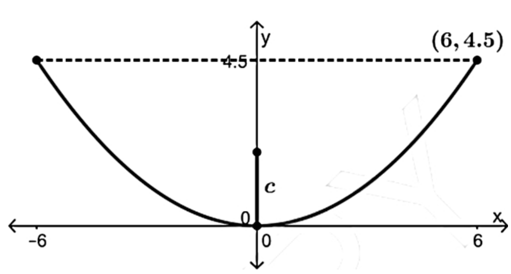
If $\sin x = \frac{\sqrt{5}}{3}$ and x lies in the 2nd quadrant, find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.





(i) Name the type of curve given in the above paragraph and find the equation of curve?

(ii) Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4).

(iii) Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x-axis. and also find the length of latus rectum.

OR

Find focus, length of latus rectum and equation of directrix of the parabola $x^2 = 8y$.

37. **Read the text carefully and answer the questions:** **[4]**

Consider the data

x_i	4	8	11	17	20	24	32
	3	5	9	5	4	3	1

f_i							

(i) Find the standard deviation.

(ii) Find the variance.

(iii) Find the mean.

OR

Write the formula of variance?

38. **Read the text carefully and answer the questions:** **[4]**

Ashish is writing examination. He is reading question paper during reading time. He reads instructions carefully. While reading instructions, he observed that the question paper consists of 15 questions divided in to two parts - part I containing 8 questions and part II containing 7 questions.

(i) If Ashish is required to attempt 8 questions in all selecting at least 3 from each part, then in how many ways can he select these questions

(ii) If Ashish is required to attempt 8 questions in all selecting 3 from I part, then in how many ways can he select these questions

Solution

Section A

1. **(a)** 2

   Explanation: Let C = 90°. Then, B = (90° - A).

   $\sin^2 A + \sin^2 B + \sin^2 C = \sin^2 A + \sin^2(90° - A) + \sin^2 90°$

   $= (\sin^2 A + \cos^2 A + 1) = 2.$

2. **(c)** $\{(x, y) : x, y \in R, y^2 = x\}$

   Explanation: A function is said to exist when we get a unique value for any value of x.

   Here, $y^2 = x$ is not a function as for each value of x, we will get 2 values of y which violates the definition of a function.

3. **(c)** P(E) = 1

   Explanation: P(E) = 1

4. **(c)** $-\frac{1}{3}$

   Explanation: Equation is in the form of $\frac{0}{0}$

   Using L'Hospital rule we get $\frac{-\frac{1}{2\sqrt{5+x}}}{\frac{1}{2\sqrt{5-x}}}$

   Substituting x = 4 we get $\frac{-1}{3}$

5. **(a)** y - 2 = 3 (x – 1)

   Explanation: Line is parallel to the line y = 3x - 1.

   Therefore, slope of the line is ‘3’.

   Also, line passes through the point (1,2).

   Thus, equation of the line is: y - 2 = 3(x - 1)

6. **(c)** B ⊆ A

   Explanation: The union of two sets is a set of all those elements that belong to A or to B or to both A and B.

   If A ∪ B = A , then B ⊆ A

7. **(a)** $\frac{1}{13}$

   Explanation: $\frac{1}{13}$

   Let z $= \frac{1}{(2+3i)^2}$

   $\Rightarrow z = \frac{1}{4+9i^2+12i}$

   $\Rightarrow z = \frac{1}{-5+12i}$

   $\Rightarrow z = \frac{1}{-5+12i} \times \frac{-5-12i}{-5-12i}$

   $\Rightarrow z = \frac{-5-12i}{25+144}$

   $\Rightarrow z = \frac{-5}{169} - \frac{12i}{169}$

   $\Rightarrow |z| = \sqrt{\frac{25}{169^2} + \frac{144}{169^2}}$

   $\Rightarrow |z| = \frac{1}{\sqrt{169}}$

   $\Rightarrow |z| = \frac{1}{13}$

8. **(c)** $(-\infty, -1) \cup (1, 4]$

   Explanation: We have, f(x)= $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

   f(x) is defined if $4 - x \geq 0$ and $x^2 - 1 > 0$

$\Rightarrow$ $x-4\leq 0$ and (x + 1)(x - 1) > 0

$\Rightarrow$ $x\leq 4$ and (x < -1 or x > 1)

$\therefore$ Domain of f $=(-\infty,-1)\cup(1,4]$

9.

(b) $-4 \leq x \leq 1$

Explanation: (x + 5) - 7(x - 2) $\geq$ 4x + 9

$\Rightarrow$ x + 5 - 7x + 14 $\geq$ 4x + 9

$\Rightarrow$ -6x + 19 $\geq$ 4x + 9

$\Rightarrow$ -6x - 4x $\geq$ 9 - 19

$\Rightarrow$ -10x $\geq$ -10

$\Rightarrow$ x $\leq$ 1

$\Rightarrow x\in(-\infty,1]$

2(x - 3) - 7(x + 5) $\leq$ 3x - 9

$\Rightarrow$ 2x - 6 - 7x - 35 $\leq$ 3x < 9

$\Rightarrow$ -5x - 41 $\leq$ 3x - 9

$\Rightarrow$ -5x - 3x $\leq$ 41 - 9

$\Rightarrow$ -8x $\leq$ 32

$\Rightarrow$ -x $\leq$ $\frac{32}{8}$ = 4

$\Rightarrow$ x $\geq$ -4

$\Rightarrow x\epsilon[-4,\infty)$

Hence the solution set is $[-4,\infty)\cap(-\infty,1]$ = [-4, 1]

Which means $-4 \leq x \leq 1$

10.

(b) sin x

Explanation: $\frac{\sin 3x}{1+2\cos 2x}$

$=\frac{3\sin x-4\sin^3 x}{1+2(1-2\sin^2 x)}$

$=\frac{\sin x(3-4\sin^2 x)}{(3-4\sin^2 x)}$ = sin x

11.

(d) {1, 2, 3, 4}

Explanation: Given A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

B $\cap$ C = {4}

A $\cup$ (B $\cap$ C) = {1, 2, 3, 4}

12.

(d) $\frac{1}{2}$

Explanation: Consider the infinite G . P $a, ar, ar^2, ar^3, \ldots\ldots\ldots\ldots\ldots,\ldots$ with first term a and common ratio r

Then the even terms $ar, ar^3, ar^5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ is again an infinite G.P with first term ar and common ratio r^2

W e have $S_\infty=\frac{a}{1-r}$

Given $S_\infty=3$. Sum of even terms

$\Rightarrow a+ar+ar^2+ar^3+$..........$=3.\left[ar+ar^3+ar^5+\right.$............]

$\Rightarrow \frac{a}{1-r}=3\cdot\frac{ar}{1-r^2}$

$\Rightarrow \frac{1}{1-r}=3\cdot\frac{r}{(1-r)(1+r)}$

$\Rightarrow 1(1+r)=3.r$

$\Rightarrow 2r=1\Rightarrow r=\frac{1}{2}$

13.

(b) an even positive integer

Explanation: We have $(a+b)^n+(a-b)^n$

$=\left[{}^nC_0a^n+{}^nC_1\quad a^{n-1}b+{}^nC_2\quad a^{n-2}b^2+{}^nC_3\quad a^{n-3}b^3+\ldots\ldots\ldots\cdots+{}^nC_n\quad b^n\right]+$

$\left[{}^nC_0\quad a^n-{}^nC_1a^{n-1}b+{}^nC_2\quad a^{n-2}b^2-{}^nC_3\quad a^{n-3}b^3+\ldots\cdots+(-1)^n\cdot{}^nC_n\right]^n$

$=2\left[{}^nC_0\quad a^n+{}^nC_2\quad a^{n-2}b^2+\ldots\ldots\ldots\ldots\right]$

Let $a=\sqrt{3}$ and b=1 and n=2n

$(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n}=2\left[{}^{2n}C_0(\sqrt{3})^{2n}+{}^{2n}C_2(\sqrt{3})^{2n-2}1^2+{}^{2n}C_4(\sqrt{3})^{2n-4}1^4+\ldots\ldots\right]$

$=2\left[{}^{2n}C_0 \quad (3)^n+{}^{2n}C_2 \quad (3)^{n-1}+{}^{2n}C_4 \quad (3)^{n-2}+\ldots\ldots\ldots\ldots\right]$

=2(a positive integer)

Hence we have $(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n}\}$ is always an even positive integer

14. **(a)** -x > -7

Explanation: x < 7

We know that when we change the sign of inequalities then greater tan changes to less than and vice versa also true.

$\Rightarrow$ -x > -7

15.

(b) A

Explanation: $(A\cap B')=A$

$\Rightarrow A\cap(A\cap B')=A\cap A=A$

16. **(a)** 10° 30'

Explanation: Speed = 90 km/hr $=\left(90\times\frac{5}{18}\right)\mathrm{m/sec}$ = 25 m/sec

Distance moved in 11 sec $=(25\times 11)\mathrm{m}$ = 275 m.

$\therefore$ l = 275 m and r = 1500 m

$\theta=\frac{l}{r}=\left(\frac{275}{1500}\right)^c=\left(\frac{275}{1500}\times\frac{180}{\pi}\right)^\circ=\left(33\times\frac{7}{22}\right)^\circ=\left(\frac{21}{2}\right)^\circ$ = 10° 30'.

17. **(a)** $\frac{(a^2+1)^4}{4a^2+1}$

Explanation: $\frac{(a^2+1)^4}{4a^2+1}$

x + iy $=\frac{(a^2+1)}{2a-i}$

Taking modulus on both the sides, we get:

$\sqrt{x^2+y^2}=\frac{(a^2+1)^2}{\sqrt{4a^2+1}}$

$x^2+y^2=\frac{(a^2+1)^4}{4a^2+1}$

18.

(d) ${}^{16}C_9$

Explanation: Total number of players = 22

2 players are always included and 4 are always excluding or never included = 22 - 2 - 4 = 16

$\therefore$ Required number of selection = ${}^{16}C_9$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1+x)^n=n_{c_0}+n_{c_1}x+n_{c_2}x^2\ldots+n_{c_n}x^n$

Reason:

$(1+(-1))^n=n_{c_0}1^n+n_{c_1}(1)^{n-1}(-1)^1+n_{c_2}(1)^{n-2}(-1)^2+\ldots+{}^nc_n(1)^{n-n}(-1)^n$

$=n_{c_8}-n_{c_1}+n_{c_2}-n_{c_3}+\ldots(-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1+(-1))^n=0$

Reason is also the but not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Let $\bar{x}$ be the mean of $x_1, x_2, \ldots, x_n$. Then, variance is given by

$\sigma_1^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$

If a is added to each observation, the new observations will be

$y_i=x_i+a$

Let the mean of the new observations be $\bar{y}$.

Then,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$$

i.e. $\bar{y} = \bar{x} + a$ ...(ii)

Thus, the variance of the new observations is $\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$ [using Eqs. (i) and (ii)]

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$$

Thus, the variance of the new observations is same as that of the original observations.

Reason: We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

Section B

21. Given, $f(x) = \frac{1}{\sqrt{x+|x|}}$

As we know, $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

$\therefore \quad x + |x| = \begin{cases} x + x, \text{ if } x \geq 0 \\ x - x, \text{ if } x < 0 \end{cases}$

$\Rightarrow \quad x + |x| = \begin{cases} 2x, \text{ if } x \geq 0 \\ 0, \text{ if } x < 0 \end{cases}$ .... (i)

The function $f(x) = \frac{1}{\sqrt{x+|x|}}$ assumes real values, if $x + |x| > 0$.

$\Rightarrow \quad 2x > 0 \Rightarrow \quad x > 0$ [using Eq. (i)]

$\therefore \quad x \in (0, \infty)$

$\therefore$ domain of $f = (0, \infty)$.

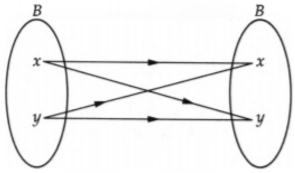
OR

According to the question,

We have, B = {x, y}

∴ B × B = (x, y) × (x, y) = {(x, x), (x, y), (y, x), (y, y)}

Following is the arrow diagram representing B × B.



22. Let u = $(x^2 - 1)$ and v = $(x^2 + 7x + 1)$

$u' = \frac{du}{dx} = \frac{d(x^2-1)}{dx} = 2x$

$v' = \frac{dv}{dx} = \frac{d(x^2+7x+1)}{dx} = 2x + 7$

Put the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{(x^2-1)}{(x^2+7x+1)}\right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3+14x^2+2x-2x^3-7x^2+2x+7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2+4x+7}{(x^2+7x+1)^2}$$

23. Let (h, k) be the centre of the circle and r be the radius of the circle.

the circle touching the lines x + 2y = 0 and x - 2 y = 0.

We know that the length of the perpendicular from the centre of a circle on the tangent line is equal to the radius of the circle.

(Length of the perpendicular from (h, k) on x + 2y = 0) = r = (Length of the perpendicular from (h, k) on x - 2y = 0)

$\Rightarrow \frac{|h+2k|}{\sqrt{1+2^2}} = r$ and , $\left|\frac{h-2k}{\sqrt{1^2+1,2^2}}\right| = r$

$\Rightarrow \frac{|h+2k|}{\sqrt{5}} = r$ and , $\frac{|h-2k|}{\sqrt{5}} = r$

$\Rightarrow \frac{|h+2k|}{\sqrt{5}} = \frac{|h-2k|}{\sqrt{5}}$

$\Rightarrow |h+2k| = |h-2k|$

$\Rightarrow$ h + 2k = ± (h - 2k)

$\Rightarrow h + 2k = h - 2k$ or $_i h + 2k = -(h-2k)$

$\Rightarrow$ 4k = 0 or, 2h = 0

$\Rightarrow$ h = 0 or, k = 0

Hence, the locus of (h, k) is x = 0 or y = 0

OR

Here we are given that $y^2 = 4ax$ is passing through the point (3, 2).

Hence this point will satisfy the equation of the parabola.

$\therefore 2^2 = 4 (a) (3)$

$\Rightarrow a = \frac{1}{3}$

Length of the latus ractum is given by

4a

$= 4 \times \frac{1}{3}$

$= \frac{4}{3}$

24. Here, we have $X \subset A$ and $X \not\subset B$

$\Rightarrow$ X is a subset of A but X is not a subset of B

$\Rightarrow X \in P(A)$ but $X \notin P(B)$,we get

$\Rightarrow$ X = {d}, {a, b, d}, {b, c, d}, {a, c, d}, {a, d}, {b, d}, {c, d}, {a, b, c, d}.

25. Given equation is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow$ bx + ay = ab

$\Rightarrow$ ay = -bx + ab

$\Rightarrow y = -\frac{b}{a}x + b$

Which is the slope intercept form of the given line.

$\therefore$ Slope = m = $-\frac{b}{a}$ and y-intercept = b

Section C

26. Clearly, R_1 and R_2 are subsets of $X \times X$. In order to prove that $R_1 = R_2$, it is sufficient to show that $R_1 \subset R_2$ and $R_2 \subset R_1$.

We observe that the difference between any two elements of each of the sets {1, 4, 7}, {2, 5, 8} and {3, 6, 9} is a multiple of 3.

Let (x, y) be an arbitrary element of R_1. Then,

$(x, y) \in R_1$

$\Rightarrow$ x - y is divisible by 3.

$\Rightarrow$x - y is a multiple of 3

$\Rightarrow$ {x, y) $\subset$ {1, 4, 7} or {x, y} $\subset$ {2, 5, 8) or (x, y) $\subset$ {3, 6, 9}

$\Rightarrow (x, y) \in R_2$

Thus, $(x, y) \in R_1 \Rightarrow (x, y) \in R_2$.

So, $R_1 \subset R_2$ ...(i)

Now, let (a, b) be an arbitrary element of R_2. Then,

$(a, b) \in R_2$

$\Rightarrow$ {a, b} $\subset$ {1, 4, 7} or {a, b} $\subset$ {2, 5, 8} or {a, b} $\subset$ {3, 6 , 9}

$\Rightarrow$ a - b is divisible by 3

$\Rightarrow (a, b) \in R_1$

Thus, $(a, b) \in R_2 \Rightarrow (a, b) \in R_1$

So, $R_2 \subset R_1$ ...(ii)

From (i) and (ii), we get: $R_1 = R_2$.

27. Case I: When $x \geq 0$

Then, $|x| = x$ and so $\frac{1}{2-|x|} \geq 1$

$\Rightarrow \frac{1}{2-x} - 1 \geq 0 \Rightarrow \frac{x-1}{2-x} \geq 0$

$\therefore$ ($x - 1 \geq 0$ and $2 - x > 0$) or ($x - 1 \leq 0$ and $2 - x < 0$)

$\Rightarrow$ ($x \geq 1$ and $x < 2$) or ($x \leq 1$ and $x > 2$)

$\Rightarrow (1 \leq x < 2)$

$\Rightarrow x \in (1, 2)$

Case II: When $x < 0$

Then, $|x| = -x$

so, $\frac{1}{2-|x|} \geq 1 \Rightarrow \frac{1}{2+x} - 1 \geq 0$

$\Rightarrow \frac{-1-x}{2+x} \geq 0 \Rightarrow \frac{1+x}{2+x} \leq 0$

$\Rightarrow$ ($1 + x \leq 0$ and $2 + x > 0$) or ($1 + x \geq 0$ and $2 + x > 0$)

$\Rightarrow (-2 < x \leq -1) \Rightarrow x \in (-2, -1)$

$\therefore$ solution set = $(-2, -1) \cup (1, 2)$

28. To prove: Points A, B, C form equilateral triangle.

Formula: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$(x_1, y_1, z_1) = (1, -1, -5)$

$(x_2, y_2, z_2) = (3, 1, 3)$

$(x_3, y_3, z_3) = (9, 1, -3)$

Length AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(3-1)^2 + (1-(-1))^2 + (3-(-5))^2}$

$= \sqrt{(2)^2 + (2)^2 + (8)^2}$

$= \sqrt{4 + 4 + 64}$

Length AB $= \sqrt{72} = 6\sqrt{2}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(9-3)^2 + (1-1)^2 + (-3-3)^2}$

$= \sqrt{(6)^2 + (0)^2 + (-6)^2}$

$= \sqrt{36 + 0 + 36}$

Length BC $= \sqrt{72} = 6\sqrt{2}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(9-1)^2 + (1-(-1))^2 + (-3-(-5))^2}$

$= \sqrt{(8)^2 + (2)^2 + (2)^2}$

$= \sqrt{64 + 4 + 4}$

Length AC $= \sqrt{72} = 6\sqrt{2}$

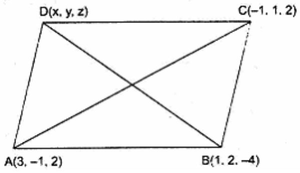
Hence, AB = BC = AC

Therefore, Points A, B, C make an equilateral triangle.

OR

Let D (x, y, z) be the fourth vertex of parallelogram ABCD.

We know that diagonals of a parallelogram bisect each other. So the mid points of AC and BD coincide.



$\therefore$ Coordinates of mid point of AC $\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$

= (1, 0, 2)

Also coordinates of mid point of BD $\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

$\therefore \frac{x+1}{2} = 1 \Rightarrow x + 1 = 2 \Rightarrow x = 1$

$\frac{y+2}{2}=0 \Rightarrow$ y + 2= 0 ⇒ y = -2

$\frac{z-4}{2}=2 \Rightarrow$ z - 4 = 4 ⇒ z = 8

Thus the coordinates of point D are (1, -2, 8)

29. Using binomial theorem for the expansion of $\left(x+\frac{1}{x}\right)^6$ we have

$\left(x+\frac{1}{x}\right)^6 = ={}^6C_0(x)^6+{}^6C_1(x)^5\left(\frac{1}{x}\right)+{}^6C_2(x)^4\left(\frac{1}{x}\right)^2+{}^6C_3(x)^3\left(\frac{1}{x}\right)^3$

$+{}^6C_4(x)^2\left(\frac{1}{x}\right)^4+{}^6C_5(x)\left(\frac{1}{x}\right)^5+{}^6C_6\left(\frac{1}{6}\right)^6$

$=x^6+6\cdot x^5\cdot\frac{1}{x}+15\cdot 4x^4\cdot\frac{1}{x^2}+20\cdot x^3\cdot\frac{1}{x^3}+15\cdot x^2\cdot\frac{1}{x^4}+6\cdot x\cdot\frac{1}{x^5}+\frac{1}{x^6}$

$=x^6+6x^4+15x^2+20+\frac{15}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$

OR

Using binomial theorem for the expansion of $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ we have

$\left(\frac{2}{x}-\frac{x}{2}\right)^5={}^5C_0\left(\frac{2}{x}\right)^5+{}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right)+{}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2+{}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3$

$+{}^5C_4\left(\frac{2}{x}\right)\left(\frac{-x}{2}\right)^4+{}^5C_5\left(\frac{-x}{2}\right)^5$

$=\frac{32}{x^5}+5\cdot\frac{16}{x^4}\cdot\frac{-x}{2}+10\cdot\frac{8}{x^3}\cdot\frac{x^2}{4}+10\cdot\frac{4}{x^2}\cdot\frac{-x^3}{8}+5\cdot\frac{2}{x}\cdot\frac{x^4}{16}+\frac{-x^5}{32}$

$=\frac{32}{x^5}-\frac{40}{x^3}+\frac{20}{x}-5x+\frac{5}{8}x^3-\frac{x^5}{32}$

30. Let $x+yi=\sqrt{1-i}$

Squaring both sides, we get

$x^2 - y^2 + 2xyi = 1 - i$

Equating the real and imaginary parts

$x^2 - y^2 = 1$ and $2xy=-1$... . (i)

$\therefore\ xy=\frac{-1}{2}$

Using the identity

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$=(1)^2+44\left(-\frac{1}{2}\right)^2$

= 1 + 1

= 2

$\therefore\ x^2+y^2=\sqrt{2}$ . . . . (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2=\frac{\sqrt{2}+1}{2}$ and $y=\frac{\sqrt{2}-1}{2}$

$\therefore x=\pm\sqrt{\frac{\sqrt{2}+1}{2}}$ and $y=\pm\sqrt{\frac{\sqrt{2}-1}{2}}$

Since the sign of xy is negative.

$\therefore$ if $x=\sqrt{\frac{\sqrt{2}+1}{2}}$ then $y=-\sqrt{\frac{\sqrt{2}-1}{2}}$

and if $x=-\sqrt{\frac{\sqrt{2}+1}{2}}$ then $y=\sqrt{\frac{\sqrt{2}-1}{2}}$

$\therefore\sqrt{1-i}=\pm\left(\sqrt{\frac{\sqrt{2}+1}{2}}-\sqrt{\frac{\sqrt{2}-1}{2}}i\right)$

OR

$\left[\frac{1}{1-4i}-\frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{1+i-2+8i}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$

$=\left[\frac{-1+9i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$

$=\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}=\frac{33+31i}{28-10i}\times\frac{28+10i}{28+10i}$

$=\frac{924+330i+868i+310i^2}{(28)^2-(10i)^2}=\frac{614+1198i}{784+100}(\because i^2=-1)$

$=\frac{2(307+599i)}{884}=\frac{307+599i}{442}$

31. Let S denote the universal set consisting of individuals suffering from the skin disorder, A denote the set of individuals exposed to chemical C_1 and B denote the set of individuals exposed to chemical C_2.

Now,

n(S) = 200

n(A) = 120

n(B) = 50

and n (A ∩ B) = 30

i. Chemical C_1 but not chemical C_2

Number of individuals exposed to chemical C_1 but not chemical C_2 is

$= n(A \cap B')$

= n(A) - $n(A \cap B)$

= 120 - 30 = 90

ii. Number of individuals exposed to chemical C_2 but not chemical C_1

$= n(A' \cap B)$

= n(B) - n(A ∩ B)

= 50 - 30 = 20

iii. Number of individuals exposed to chemical C_1 or chemical C_2

$= n(A \cup B)$

= n(A) + n(B) - n(A ∩ B)

= 120 + 50 - 30

= 140

Section D

32. Given that the bag contains 13 balls and three balls are drawn from the bag

So, the total number of ways of drawing three balls = number of total outcomes $=n(S) = {}^{13}C_3$

Now, we have to find the probability that all three balls drawn are red,

Let A be the event that all drawn balls are red

There are 8 red balls in the bag

So, number of favourable outcomes i.e. all three balls are red = n(A) = ${}^{8}C_3$

We know that,

P(A) = $\frac{\mathbf{n(A)}}{\mathbf{n(S)}} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

P(all the three balls are red) = $\frac{{}^{8}C_3}{{}^{13}C_3}$

$= \frac{\frac{8!}{3!(8-3)!}}{\frac{13!}{3!(13-3)!}} \left[\because {}^{n}C_r = \frac{n!}{(n-r)!r!}\right]$

$= \frac{\frac{8\times7\times6\times5!}{3!5!}}{\frac{13\times12\times11\times10!}{3\times2\times1\times10!}}$

$= \frac{\frac{8\times7\times6}{3\times2}}{13\times2\times11}$

$= \frac{28}{143}$

33. The given function is,

f(x) = cos(x^2 + 1) ...(i)

Taking increment,we have,

$\Rightarrow \quad f(x+\Delta x) = \cos\left[(x+\Delta x)^2+1\right]$ ...(ii)

Subtracting eq. (i) from eq. (ii) we get

$f(x+\Delta x) - f(x) = \cos\left[(x+\Delta x)^2+1\right] - \cos\left(x^2+1\right)$

Dividing both sides by Δx we have,

$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\cos\left[(x+\Delta x)^2+1\right]-\cos\left(x^2+1\right)}{\Delta x}$

$\Rightarrow \lim\limits_{\Delta x\to0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim\limits_{\Delta x\to0} \frac{\cos\left[(x+\Delta x)^2+1\right]-\cos\left(x^2+1\right)}{\Delta x}$

$f'(x) = \lim\limits_{\Delta x\to0} \frac{\cos\left[(x+\Delta x)^2+1\right]-\cos\left(x^2+1\right)}{\Delta x}$ [using definition of derivative]

$-2\sin\left[\frac{(x+\Delta x)^2+1+x^2+1}{2}\right]$

$= \lim\limits_{\Delta x\to0} \frac{\sin\left[\frac{(x+\Delta x)^2+1-x^2-1}{2}\right]}{\Delta x}$ $\left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\cdot\sin\frac{C-D}{2}\right]$

$$= \lim_{\Delta x \to 0} \frac{-2\sin\left[\frac{x^2+\Delta x^2+2x\cdot\Delta x+x^2+2}{2}\right]\cdot\sin\left[\frac{x^2+\Delta x^2+2x\Delta x-x^2}{2}\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\sin\left[x^2+\frac{\Delta x^2}{2}+x\Delta x+1\right]\sin\left[\Delta x\frac{(\Delta x+2x)}{2}\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\sin\left[x^2+\frac{\Delta x^2}{2}+x\Delta x+1\right]\cdot\sin\left[\Delta x\frac{(\Delta x+2x)}{2}\right]}{\Delta x\left[\frac{\Delta x+2x}{2}\right]} \times \left(\frac{\Delta x+2x}{2}\right)$$

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta x\left[\frac{\Delta x+2x}{2}\right] \to 0}} -2\sin\left[x^2+\frac{\Delta x^2}{2}+x\Delta x+1\right]\cdot\frac{\sin\left[\Delta x\frac{(\Delta x+2x)}{2}\right]}{\Delta x\left[\frac{\Delta x+2x}{2}\right]} \times \left[\frac{\Delta x+2x}{2}\right]$$

Taking limits, we get,

$$= -2\sin\left(x^2+1\right)\cdot 1\cdot(x) = -2x\sin\left(x^2+1\right)\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

OR

We have to find the value $\lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-(\sqrt{3}+\sqrt{2})}{x^2-6}$

Re-writing the equation as

$$= \lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{(\sqrt{3}+\sqrt{2})^2}}{x^2-6}$$

$$= \lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{3+2+2\sqrt{6}}}{x^2-6}$$

$$= \lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{5+2\sqrt{6}}}{x^2-6}$$

Now rationalizing the above equation

$$= \lim_{x\to\sqrt{6}}\frac{(\sqrt{5+2x}-\sqrt{5+2\sqrt{6}})}{x^2-6}\frac{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$$

Formula: $(a + b)(a - b) = a^2 - b^2$

$$= \lim_{x\to\sqrt{6}}\frac{(5+2x-(5+2\sqrt{6}))}{x^2-6}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$$

$$= \lim_{x\to\sqrt{6}}\frac{(2x-2\sqrt{6})}{x^2-6}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$$

$$= \lim_{x\to\sqrt{6}}\frac{2(x-\sqrt{6})}{(x+\sqrt{6})(x-\sqrt{6})}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$$

$$= \lim_{x\to\sqrt{6}}\frac{2(1)}{(x+\sqrt{6})(1)}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$$

$$= \frac{2}{2\sqrt{6}}\frac{1}{(2\sqrt{5+2\sqrt{6}})}$$

$$= \frac{1}{2\sqrt{6}}\frac{1}{(\sqrt{5+2\sqrt{6}})}$$

34. $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$

$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

by C and D

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

Sq both side

$\frac{a}{b} = \frac{m+n+m-n+2\sqrt{m^2-n^2}}{m+n+m-n-2\sqrt{m^2-n^2}}$

$\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

35. We have to prove that cos 36^o cos 42^o cos 60^o cos 78^o = $\frac{1}{16}$.

LHS = cos 36^o cos 42^o cos 60^o cos 78^o

By regrouping the LHS and multiplying and dividing by 2 we get,

$=\frac{1}{2}\cos 36^o \cos 60^o (2\cos 78^o \cos 42^o)$

But 2 cos A cos B = cos (A + B) + cos (A - B)

Then the above equation becomes,

$=\frac{1}{2}\cos 36^o \cos 60^o (\cos(78^o + 42^o) + \cos(48^o - 42^o))$

$=\frac{1}{2}\cos 36^o \cos 60^o (\cos(120^o) + \cos(36^o))$

$=\frac{1}{2}\cos 36^o \cos 60^o (\cos(180^o - 60^o) + \cos(36^o))$

But $\cos(90^o - \theta) = \sin\theta$ and $\cos(180^o - \theta) = -\cos(\theta)$.

Then the above equation becomes,

$=\frac{1}{2}\cos 36^o \cos 60^o (-\cos(60^o) + \cos(36^o))$

Now, $\cos(36^o) = \frac{\sqrt{5}+1}{4}$

$\cos(60^o) = \frac{1}{2}$

Substituting the corresponding values, we get

$=\frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right)$

$=\left(\frac{\sqrt{5}+1}{16}\right)\left(\frac{\sqrt{5}+1-2}{4}\right)$

$=\left(\frac{(\sqrt{5})^2-1^2}{16\times 4}\right)$

$=\left(\frac{5-1}{64}\right)$

$\frac{1}{16}$

LHS = RHS

Hence proved.

OR

We know,

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \pm\sqrt{1-\frac{5}{9}} = \pm\frac{2}{3}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right)$

cos x will be negative in second quadrant

therefore. $\cos x = -\frac{2}{3}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\left(\frac{-2}{3}\right)}{2}} = \pm\sqrt{\frac{1}{6}}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\cos\frac{x}{2}$ will be positive in 1st quadrant.

So, $\cos\frac{x}{2} = \frac{1}{\sqrt{6}}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 1 - 2\sin^2\frac{x}{2}$ ... $[\because \cos x = -\frac{2}{3}]$

$-\frac{2}{3} = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = \frac{2}{3}+1$

$2\sin^2\frac{x}{2} = \frac{2+3}{3}$

$\sin^2\frac{x}{2} = \frac{5}{6}$

$\sin^2\frac{x}{2} = \pm\sqrt{\frac{5}{6}}$

Since, x ∈ $\left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\sin\frac{x}{2}$ will be positive in 1st quadrant

So,

$\sin\frac{x}{2} = \sqrt{\frac{5}{6}}$

We know,

$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$\tan\frac{x}{2} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}}$

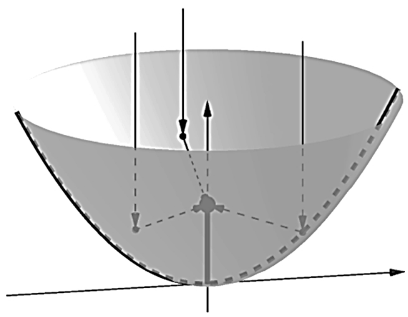
$\tan\frac{x}{2} = \sqrt{5}$

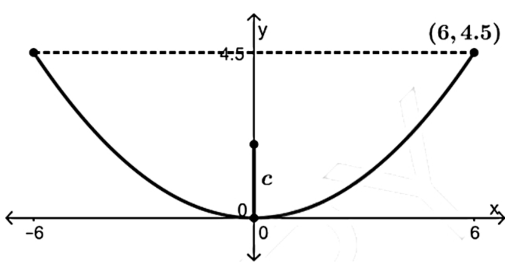
Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$, $\tan\frac{x}{2}$ are $\frac{1}{\sqrt{6}}$, $\sqrt{\frac{5}{6}}$ and $\sqrt{5}$

Section E

36. **Read the text carefully and answer the questions:**

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.





(i) Given curve is a parabola

Equation of parabola is $x^2 = 4ay$

It passes through the point (6, 4.5)

$\Rightarrow 36 = 4 \times a \times 4.5$

$\Rightarrow 36 = 18a$

$\Rightarrow a = 2$

Equation of parabola is $x^2 = 8y$

(ii) Distance between focus and vertex is = a = $\sqrt{(4-4)^2 + (5-3)^2} = 2$

Equation of parabola is $(y - k)^2 = 4a(x - h)$

where (h, k) is vertex

⇒ Equation of parabola with vertex (3, 4) & a = 2

$\Rightarrow (y - 4)^2 = 8(x - 3)$

(iii) Equation of parabola with axis along x - axis

$y^2 = 4ax$

which passes through (2, 3)

$\Rightarrow 9 = 4a \times 2$

$\Rightarrow 4a = \frac{9}{2}$

hence required equation of parabola is

$y^2 = \frac{9}{2}x$

$\Rightarrow 2y^2 = 9x$

Hence length of latus rectum = 4a = 4.5

OR

$x^2 = 8y$

a = 2

Focus of parabola is (0, 2)

length of latus rectum is 4a = 4 × 2 =8

Equation of directrix y + 2 =0

37. **Read the text carefully and answer the questions:**

Consider the data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

(i) By using formula,

$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{n} f_i(x_i - \bar{x})^2\right]$

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	420			1374

Given, N = $\sum f_i$ = 30, $\sum f_i x_i$ = 420 and $\sum f_i(x_i - \bar{x})^2$ = 1374

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N}$ = $\frac{420}{30}$ = 14

Variance (σ^2) = $\frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$ = $\frac{1}{30} \times 1374 = 45.8$

Standard deviation, σ = $\sqrt{\sigma^2}$ = $\sqrt{45.8}$ = 6.77

(ii) Variance (σ^2) = $\frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$ = $\frac{1}{30} \times 1374 = 45.8$

(iii) Given, N = $\sum f_i$ = 30, $\sum f_i x_i$ = 420 and $\sum f_i(x_i - \bar{x})^2$ = 1374

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N}$ = $\frac{420}{30}$ = 14

OR

$\sigma^2 = \frac{1}{N}\Sigma(x_i - \bar{x})$

38. **Read the text carefully and answer the questions:**

Ashish is writing examination. He is reading question paper during reading time. He reads instructions carefully. While reading instructions, he observed that the question paper consists of 15 questions divided in to two parts - part I containing 8 questions and part II containing 7 questions.

(i) Since, at least 3 questions from each part have to be selected

Part I	Part II
3	5
4	4
3	5

So number of ways are

3 questions from part I and 5 questions from part II can be selected in $n^{8}C_3 \times {}^{7}C_5$ ways

4 questions from part I and 4 questions from part II can be selected in ${}^{8}C_4 \times {}^{7}C_4$ ways

5 questions from part I and 3 questions from part II can be selected in ${}^{8}C_5 \times {}^{7}C_3$ ways

So required number of ways are

${}^{8}C_3 \times {}^{7}C_5 + {}^{8}C_4 \times {}^{7}C_4 + {}^{8}C_5 \times {}^{7}C_3$

$\Rightarrow \frac{8!}{5!\times 3!} \times \frac{7!}{5!\times 2!} + \frac{8!}{4!\times 4!} \times \frac{7!}{4!\times 3!} + \frac{8!}{5!\times 3!} \times \frac{7!}{4!\times 3!}$

$\Rightarrow \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6}{2\times 1} + \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1} \times \frac{7\times 6\times 5}{3\times 2\times 1} + \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1}$

$\Rightarrow$ 56 $\times$ 21 + 70 $\times$ 35 + 56 $\times$ 35

$\Rightarrow$ 1344 + 2450 + 1960

$\Rightarrow$ 5754

(ii) Ashish is selecting 3 questions from part I so he has to select remaining 5 questions from part II

The number of ways of selection is

3 questions from part I and 5 questions from part II can be selected in ${}^{8}C_3 \times {}^{7}C_5$ ways

$\Rightarrow {}^{8}C_3 \times {}^{7}C_5$

$\Rightarrow \frac{8!}{5!\times 3!} \times \frac{7!}{5!\times 2!}$

$\Rightarrow \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6}{2\times 1}$

$\Rightarrow 56 \times 21$

$\Rightarrow 1344$